**Closed-Loop Model Equations**

To set up an averaging calculation to obtain the approximate dynamics of the control variable, $y = P_{\rm a}O_2$, we write the closed-loop control ODE model in the following form:

$$\begin{aligned}
\text{Voltage, BRS cell.} \quad & \frac{dV}{dt} = f_V(V, h, n, y) && (1)\\
\text{h-gate (NaP), BRS cell.} \quad & \frac{dh}{dt} = f_h(V, h) && (2)\\
\text{n-gate (K), BRS cell.} \quad & \frac{dn}{dt} = f_n(V, n) && (3)\\
\text{Motor pool activity.} \quad & \frac{d\alpha}{dt} = f_\alpha(V, \alpha) && (4)\\
\text{Lung volume.} \quad & \frac{dL}{dt} = f_L(\alpha, L) && (5)\\
\text{PO}_2\text{(lung).} \quad & \frac{dO}{dt} = f_O(L, O, y) && (6)\\
\text{PO}_2\text{(blood).} \quad & \frac{dy}{dt} = g(O, y) && (7)
\end{aligned}$$

For the reasons articulated in the manuscript, we perform averaging with respect to $P_{\rm a}O_2$, and consider equations (1-6) to be the "fast" subsystem and equation (7) to be the "slow" subsystem.