Details of Rubin and Terman's model

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1 Details of Rubin and Terman's model

In this report we describe the mathematical model extracted from an XPPAUT file provided by David Terman. Each cell type is modeled with a single-compartment conductance-based biophysical Hodgkin-Huxley-like model. Every modification we made to the original file in order to perform our own simulations is indicated explicitely.

1.1 Differential equations associated to each cell model

For clarity, dependences of ion currents on membrane voltage and gating variables are omitted. See subsection 1.2 of this appendix for the details of these dependences.

1.1.1 Differential equations for STN cells

- v is the *voltage of STN neurons.*

$\forall j \in \{1, \ldots, 8\}$:

$$\frac{d}{dt}v_M^{STN,j} = \frac{-i_L^{STN,j}-i_{Na}^{STN,j}-i_K^{STN,j}-i_{AHP}^{STN,j}-i_{Ca}^{STN,j}-i_T^{STN,j}-i_{GPe\to STN}^{STN,j}+jI_0^{STN}+i_{HFS}^{STN,j}}{C_M^{STN}}$$

$$\frac{d}{dt}h^{STN,j} = \phi_h^{STN}\frac{h_\infty^{STN}\left(v_M^{STN,j}\right)-h^{STN,j}}{\tau_h^{STN}\left(v_M^{STN,j}\right)}$$

$$\frac{d}{dt}n^{STN,j} = \phi_n^{STN}\frac{n_\infty^{STN}\left(v_M^{STN,j}\right)-n^{STN,j}}{\tau_n^{STN}\left(v_M^{STN,j}\right)}$$

$$\frac{d}{dt}r^{STN,j} = \phi_r^{STN}\frac{r_\infty^{STN}\left(v_M^{STN,j}\right)-r^{STN,j}}{\tau_r^{STN}\left(v_M^{STN,j}\right)}$$

$$\frac{d}{dt}x_{Ca}^{STN,j} = \phi_{x_{Ca}}^{STN}\epsilon_{x_{Ca}}^{STN}\left(-i_{Ca}^{STN,j}-i_T^{STN,j}-k_{Ca}^{STN}x_{Ca}^{STN,j}\right)$$

$$\frac{d}{dt}s^{STN,j} = \alpha^{STN}\left(1-s^{STN,j}\right)H_\infty^{STN}\left(v_M^{STN,j}-\theta_g^{STN}\right)-\beta^{STN}s^{STN,j}$$

1.1.2 Differential equations for GPe cells

$\forall j \in \{1, \ldots, 8\}$:

$$\frac{d}{dt}v_M^{GPe,j} = \frac{-i_L^{GPe,j}-i_{Na}^{GPe,j}-i_K^{GPe,j}-i_{AHP}^{GPe,j}-i_{Ca}^{GPe,j}-i_T^{GPe,j}-i_{GPe\to GPe}^{GPe,j}-i_{STN\to GPe}^{GPe,j}+jI_0^{GPe}+I_{app}^{GPe}}{C_M^{GP}}$$

$$\frac{d}{dt}h^{GPe,j} = \phi_h^{GP}\frac{h_\infty^{GP}\left(v_M^{GPe,j}\right)-h^{GPe,j}}{\tau_h^{GP}\left(v_M^{GPe,j}\right)}$$

$$\frac{d}{dt}n^{GPe,j} = \phi_n^{GP}\frac{n_\infty^{GP}\left(v_M^{GPe,j}\right)-n^{GPe,j}}{\tau_n^{GP}\left(v_M^{GPe,j}\right)}$$

$$\frac{d}{dt}r^{GPe,j} = \phi_r^{GP}\frac{r_\infty^{GP}\left(v_M^{GPe,j}\right)-r^{GPe,j}}{\tau_r^{GP}}$$
$$\frac{d}{dt}x_{Ca}^{GPe,j} = \epsilon_{x_{Ca}}^{GP}\left(-i_{Ca}^{GPe,j}-i_T^{GPe,j}-k_{Ca}^{GP}x_{Ca}^{GPe,j}\right)$$
$$\frac{d}{dt}s^{GPe,j} = \alpha^{GP}\left(1-s^{GPe,j}\right)H_\infty^{GP}\left(v_M^{GPe,j}-\theta_g^{GP}\right)-\beta^{GPe}s^{GPe,j}$$

1.1.3 Differential equations for GPi cells

$\forall j \in \{1,\ldots,8\}$:

$$\frac{d}{dt}v_M^{GPi,j} = \frac{-i_L^{GPi,j}-i_{Na}^{GPi,j}-i_K^{GPi,j}-i_{AHP}^{GPi,j}-i_{Ca}^{GPi,j}-i_T^{GPi,j}-i_{STN\to GPi}^{GPi,j}+I_{app}^{GPi}}{C_M^{GP}}$$
$$\frac{d}{dt}h^{GPi,j} = \phi_h^{GP}\frac{h_\infty^{GP}\left(v_M^{GPi,j}\right)-h^{GPi,j}}{\tau_h^{GP}\left(v_M^{GPi,j}\right)}$$
$$\frac{d}{dt}n^{GPi,j} = \phi_n^{GP}\frac{n_\infty^{GP}\left(v_M^{GPi,j}\right)-n^{GPi,j}}{\tau_n^{GP}\left(v_M^{GPi,j}\right)}$$
$$\frac{d}{dt}r^{GPi,j} = \phi_r^{GP}\frac{r_\infty^{GP}\left(v_M^{GPi,j}\right)-r^{GPi,j}}{\tau_r^{GP}}$$
$$\frac{d}{dt}x_{Ca}^{GPi,j} = \epsilon_{x_{Ca}}^{GP}\left(-i_{Ca}^{GPi,j}-i_T^{GPi,j}-k_{Ca}^{GP}x_{Ca}^{GPi,j}\right)$$
$$\frac{d}{dt}s^{GPi,j} = \alpha^{GP}\left(1-s^{GPi,j}\right)H_\infty^{GP}\left(v_M^{GPi,j}-\theta_g^{GP}\right)-\beta^{GPi}s^{GPi,j}$$

1.1.4 Differential equations for thalamic cells

$\forall j \in \{1,2\}$:

$$\frac{d}{dt}v_M^{Thl,j} = \frac{-i_L^{Thl,j}-i_{Na}^{Thl,j}-i_K^{Thl,j}-i_T^{Thl,j}-i_{GPi\to Thl}^{Thl,j}+i_{SM}^{Thl,j}}{C_M^{Thl}}$$
$$\frac{d}{dt}h^{Thl,j} = \phi_h^{Thl}\frac{h_\infty^{Thl}\left(v_M^{Thl,j}\right)-h^{Thl,j}}{\tau_h^{Thl}\left(v_M^{Thl,j}\right)}$$
$$\frac{d}{dt}r^{Thl,j} = \phi_r^{Thl}\frac{r_\infty^{Thl}\left(v_M^{Thl,j}\right)-r^{Thl,j}}{\tau_r^{Thl}\left(v_M^{Thl,j}\right)}$$

1.2 Ion currents

1.2.1 STN cells' currents

$\forall j \in \{1,\ldots,8\}$:

$$i_L^{STN,j} = g_L^{STN}\left(v_M^{STN,j}-V_L^{STN}\right)$$
$$i_{Na}^{STN,j} = g_{Na}^{STN}\left(m_\infty^{STN}\left(v_M^{STN,j}\right)\right)^3 h^{STN,j}\left(v_M^{STN,j}-V_{Na}^{STN}\right)$$
$$i_K^{STN,j} = g_K^{STN}\left(n^{STN,j}\right)^4\left(v_M^{STN,j}-V_K^{STN}\right)$$
$$i_{AHP}^{STN,j} = g_{AHP}^{STN}\left(v_M^{STN,j}-V_K^{STN}\right)\frac{x_{Ca}^{STN,j}}{x_{Ca}^{STN,j}+k_1^{STN}}$$
$$i_{Ca}^{STN,j} = g_{Ca}^{STN}\left(s_\infty^{STN}\left(v_M^{STN,j}\right)\right)^2\left(v_M^{STN,j}-V_{Ca}^{STN}\right)$$
$$i_T^{STN,j} = g_T^{STN}\left(a_\infty^{STN}\left(v_M^{STN,j}\right)\right)^3\left(b_\infty^{STN}\left(r^{STN,j}\right)\right)^2\left(v_M^{STN,j}-V_{Ca}^{STN}\right)$$
$$i_{GPe\to STN}^{STN,j} = g_{GPe\to STN}\left(v_M^{STN,j}-V_{GPe\to STN}\right)s_{GPe\to STN}^{STN,j}$$
$$i_{HFS}^{STN,j} = \begin{cases} I_{HFS}^{STN}Y\left(\sin\left(\omega_{HFS}^{STN}t\right)-a_{HFS}^{STN}\right) & \text{(Terman's file)} \\ I_{HFS}^{STN}Y\left(\sin\left(\frac{2\pi t}{T_{HFS}^{STN}}\right)\right)\left(1-Y\left(\sin\left(\frac{2\pi\left(t+w_{HFS}^{STN}\right)}{T_{HFS}^{STN}}\right)\right)\right) & \text{(Our simulations)} \end{cases}$$

1.2.2 GPe cells' currents

$\forall j \in \{1, \ldots, 8\}$:
$i_L^{GPe,j} = g_L^{GP}\left(v_M^{GPe,j} - V_L^{GP}\right)$
$i_{Na}^{GPe,j} = g_{Na}^{GP}\left(m_\infty^{GP}\left(v_M^{GPe,j}\right)\right)^3 h^{GPe,j}\left(v_M^{GPe,j} - V_{Na}^{GP}\right)$
$i_K^{GPe,j} = g_K^{GP}\left(n^{GPe,j}\right)^4\left(v_M^{GPe,j} - V_K^{GP}\right)$
$i_{AHP}^{GPe,j} = g_{AHP}^{GP}\left(v_M^{GPe,j} - V_K^{GP}\right)\frac{x_{Ca}^{GPe,j}}{x_{Ca}^{GPe,j}+k_1^{GP}}$
$i_{Ca}^{GPe,j} = g_{Ca}^{GP}\left(s_\infty^{GP}\left(v_M^{GPe,j}\right)\right)^2\left(v_M^{GPe,j} - V_{Ca}^{GP}\right)$
$i_T^{GPe,j} = g_T^{GP}\left(a_\infty^{GP}\left(v_M^{GPe,j}\right)\right)^3 r^{GPe,j}\left(v_M^{GPe,j} - V_{Ca}^{GP}\right)$
$i_{GPe\to GPe}^{GPe,j} = g_{GPe\to GPe}\left(v_M^{GPe,j} - V_{GPe\to GPe}\right)s_{GPe\to GPe}^{GPe,j}$
$i_{STN\to GPe}^{GPe,j} = g_{STN\to GPe}\left(v_M^{GPe,j} - V_{STN\to GPe}\right)s_{STN\to GPe}^{GPe,j}$

1.2.3 GPi cells' currents

$\forall j \in \{1, \ldots, 8\}$:
$i_L^{GPi,j} = g_L^{GP}\left(v_M^{GPi,j} - V_L^{GP}\right)$
$i_{Na}^{GPi,j} = g_{Na}^{GP}\left(m_\infty^{GP}\left(v_M^{GPi,j}\right)\right)^3 h^{GPi,j}\left(v_M^{GPi,j} - V_{Na}^{GP}\right)$
$i_K^{GPi,j} = g_K^{GP}\left(n^{GPi,j}\right)^4\left(v_M^{GPi,j} - V_K^{GP}\right)$
$i_{AHP}^{GPi,j} = g_{AHP}^{GP}\left(v_M^{GPi,j} - V_K^{GP}\right)\frac{x_{Ca}^{GPi,j}}{x_{Ca}^{GPi,j}+k_1^{GP}}$
$i_{Ca}^{GPi,j} = g_{Ca}^{GP}\left(s_\infty^{GP}\left(v_M^{GPi,j}\right)\right)^2\left(v_M^{GPi,j} - V_{Ca}^{GP}\right)$
$i_T^{GPi,j} = g_T^{GP}\left(a_\infty^{GP}\left(v_M^{GPi,j}\right)\right)^3 r^{GPi,j}\left(v_M^{GPi,j} - V_{Ca}^{GP}\right)$

$$i_{STN\to GPi}^{GPi,j} = \begin{cases} g_{STN\to GPi}\left(v_M^{GPi,j} - V_{STN\to GPi}\right)Y\left(v_{STN\to GPi}^{GPi,j} - \theta_{STN\to GPi}\right) & \text{(Terman's file)} \\ g_{STN\to GPi}\left(v_M^{GPi,j} - V_{STN\to GPi}\right)s_{STN\to GPi}^{GPi,j} & \text{(Our simulations)} \end{cases}$$

1.2.4 Thalamic cells' currents

$\forall j \in \{1, 2\}$:
$i_L^{Thl,j} = g_L^{Thl}\left(v_M^{Thl,j} - V_L^{Thl}\right)$
$i_{Na}^{Thl,j} = g_{Na}^{Thl}\left(m_\infty^{Thl}\left(v_M^{Thl,j}\right)\right)^3 h^{Thl,j}\left(v_M^{Thl,j} - V_{Na}^{Thl}\right)$
$i_K^{Thl,j} = g_K^{Thl}\left(0.75\left(1 - h^{Thl,j}\right)\right)^4\left(v_M^{Thl,j} - V_K^{Thl}\right)$
$i_T^{Thl,j} = g_T^{Thl}\left(p_\infty^{Thl}\left(v_M^{Thl,j}\right)\right)^2 r^{Thl,j}\left(v_M^{Thl,j} - V_T^{Thl}\right)$
$i_{GPi\to Thl}^{Thl,j} = g_{GPi\to Thl}\left(v_M^{Thl,j} - V_{GPi\to Thl}\right)s_{GPi\to Thl}^{Thl,j}$
$i_{SM}^{Thl,j} = I_{SM}^{Thl}Y\left(\sin\left(\frac{2\pi\left(t-d_{SM}^{Thl}\right)}{T_{SM}^{Thl}}\right)\right)\left(1 - Y\left(\sin\left(\frac{2\pi\left(t-d_{SM}^{Thl}+w_{SM}^{Thl}\right)}{T_{SM}^{Thl}}\right)\right)\right)$

1.3 Gating functions

1.3.1 STN cells' gating functions

$m_\infty^{STN}(\cdot) = \frac{1}{1+\exp\left(-\frac{\cdot-\theta_m^{STN}}{\sigma_m^{STN}}\right)}$

$h_\infty^{STN}(\cdot) = \frac{1}{1+\exp\left(\frac{\cdot-\theta_h^{STN}}{\sigma_h^{STN}}\right)}$

$n_\infty^{STN}(\cdot) = \frac{1}{1+\exp\left(-\frac{\cdot-\theta_n^{STN}}{\sigma_n^{STN}}\right)}$

$s_\infty^{STN}(\cdot) = \frac{1}{1+\exp\left(-\frac{\cdot-\theta_s^{STN}}{\sigma_s^{STN}}\right)}$

$a_\infty^{STN}(\cdot) = \frac{1}{1+\exp\left(-\frac{\cdot-\theta_a^{STN}}{\sigma_a^{STN}}\right)}$

$b_\infty^{STN}(\cdot) = \frac{1}{1+\exp\left(-\frac{\cdot-\theta_b^{STN}}{\sigma_b^{STN}}\right)} - \frac{1}{1+\exp\left(\frac{\theta_b^{STN}}{\sigma_b^{STN}}\right)}$

$r_\infty^{STN}(\cdot) = \frac{1}{1+\exp\left(\frac{\cdot-\theta_r^{STN}}{\sigma_r^{STN}}\right)}$

$\tau_h^{STN}(\cdot) = \tau_{h0}^{STN} + \frac{\tau_{h1}^{STN}}{1+\exp\left(\frac{\cdot-\theta_{h\tau}^{STN}}{\sigma_{h\tau}^{STN}}\right)}$

$\tau_n^{STN}(\cdot) = \tau_{n0}^{STN} + \frac{\tau_{n1}^{STN}}{1+\exp\left(\frac{\cdot-\theta_{n\tau}^{STN}}{\sigma_{n\tau}^{STN}}\right)}$

$\tau_r^{STN}(\cdot) = \tau_{r0}^{STN} + \frac{\tau_{r1}^{STN}}{1+\exp\left(\frac{\cdot-\theta_{r\tau}^{STN}}{\sigma_{r\tau}^{STN}}\right)}$

$H_\infty^{STN}(\cdot) = \frac{1}{1+\exp\left(-\frac{\cdot-\theta_{gH}^{STN}}{\sigma_{gH}^{STN}}\right)}$

1.3.2 GPe and GPi cells' shared gating functions

$m_\infty^{GP}(\cdot) = \frac{1}{1+\exp\left(-\frac{\cdot-\theta_m^{GP}}{\sigma_m^{GP}}\right)}$

$h_\infty^{GP}(\cdot) = \frac{1}{1+\exp\left(\frac{\cdot-\theta_h^{GP}}{\sigma_h^{GP}}\right)}$

$n_\infty^{GP}(\cdot) = \frac{1}{1+\exp\left(-\frac{\cdot-\theta_n^{GP}}{\sigma_n^{GP}}\right)}$

$s_\infty^{GP}(\cdot) = \frac{1}{1+\exp\left(-\frac{\cdot-\theta_s^{GP}}{\sigma_s^{GP}}\right)}$

$a_\infty^{GP}(\cdot) = \frac{1}{1+\exp\left(-\frac{\cdot-\theta_a^{GP}}{\sigma_a^{GP}}\right)}$

$r_\infty^{GP}(\cdot) = \frac{1}{1+\exp\left(\frac{\cdot-\theta_r^{GP}}{\sigma_r^{GP}}\right)}$

$\tau_h^{GP}(\cdot) = \tau_{h0}^{GP} + \frac{\tau_{h1}^{GP}}{1+\exp\left(\frac{\cdot-\theta_{h\tau}^{GP}}{\sigma_{h\tau}^{GP}}\right)}$

$\tau_n^{GP}(\cdot) = \tau_{n0}^{GP} + \frac{\tau_{n1}^{GP}}{1+\exp\left(\frac{\cdot-\theta_{n\tau}^{GP}}{\sigma_{n\tau}^{GP}}\right)}$

$H_\infty^{GP}(\cdot) = \frac{1}{1+\exp\left(-\frac{\cdot-\theta_{gH}^{GP}}{\sigma_{gH}^{GP}}\right)}$

1.3.3 Thalamic cells' gating functions

$m_\infty^{Thl}(\cdot) = \frac{1}{1+\exp\left(-\frac{\cdot-\theta_m^{Thl}}{\sigma_m^{Thl}}\right)}$

$h_\infty^{Thl}(\cdot) = \frac{1}{1+\exp\left(\frac{\cdot-\theta_h^{Thl}}{\sigma_h^{Thl}}\right)}$

$p_\infty^{Thl}(\cdot) = \frac{1}{1+\exp\left(-\frac{\cdot-\theta_p^{Thl}}{\sigma_p^{Thl}}\right)}$

$r_\infty^{Thl}(\cdot) = \frac{1}{1+\exp\left(\frac{\cdot-\theta_r^{Thl}}{\sigma_r^{Thl}}\right)}$

$a_h^{Thl}(\cdot) = a_{h0}^{Thl}\exp\left(-\frac{\cdot-\theta_{ah}^{Thl}}{\sigma_{ah}^{Thl}}\right)$

$b_h^{Thl}(\cdot) = \frac{b_{h0}^{Thl}}{1+\exp\left(-\frac{\cdot-\theta_{bh}^{Thl}}{\sigma_{bh}^{Thl}}\right)}$

$\tau_h^{Thl}(\cdot) = \frac{1}{a_h^{Thl}(\cdot)+b_h^{Thl}(\cdot)}$

$\tau_r^{Thl}(\cdot) = \tau_{r0}^{Thl} + \tau_{r1}^{Thl}\exp\left(-\frac{\cdot-\theta_{r\tau}^{Thl}}{\sigma_{r\tau}^{Thl}}\right)$

1.3.4 Auxiliary sigmoidal function

$Y(\cdot) = \frac{1}{1+\exp\left(-\frac{\cdot}{\sigma_Y}\right)}$

1.4 Connections among cells

1.4.1 From GPe to STN

$$\begin{pmatrix} s_{GPe\to STN}^{STN,1} \\ s_{GPe\to STN}^{STN,2} \\ s_{GPe\to STN}^{STN,3} \\ s_{GPe\to STN}^{STN,4} \\ s_{GPe\to STN}^{STN,5} \\ s_{GPe\to STN}^{STN,6} \\ s_{GPe\to STN}^{STN,7} \\ s_{GPe\to STN}^{STN,8} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} s^{GPe,1} \\ s^{GPe,2} \\ s^{GPe,3} \\ s^{GPe,4} \\ s^{GPe,5} \\ s^{GPe,6} \\ s^{GPe,7} \\ s^{GPe,8} \end{pmatrix}$$

1.4.2 From GPe to GPe

$$\begin{pmatrix} s_{GPe\to GPe}^{GPe,1} \\ s_{GPe\to GPe}^{GPe,2} \\ s_{GPe\to GPe}^{GPe,3} \\ s_{GPe\to GPe}^{GPe,4} \\ s_{GPe\to GPe}^{GPe,5} \\ s_{GPe\to GPe}^{GPe,6} \\ s_{GPe\to GPe}^{GPe,7} \\ s_{GPe\to GPe}^{GPe,8} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} s^{GPe,1} \\ s^{GPe,2} \\ s^{GPe,3} \\ s^{GPe,4} \\ s^{GPe,5} \\ s^{GPe,6} \\ s^{GPe,7} \\ s^{GPe,8} \end{pmatrix}$$

1.4.3 From STN to GPe

$$\begin{pmatrix} s_{STN\to GPe}^{GPe,1} \\ s_{STN\to GPe}^{GPe,2} \\ s_{STN\to GPe}^{GPe,3} \\ s_{STN\to GPe}^{GPe,4} \\ s_{STN\to GPe}^{GPe,5} \\ s_{STN\to GPe}^{GPe,6} \\ s_{STN\to GPe}^{GPe,7} \\ s_{STN\to GPe}^{GPe,8} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} s^{STN,1} \\ s^{STN,2} \\ s^{STN,3} \\ s^{STN,4} \\ s^{STN,5} \\ s^{STN,6} \\ s^{STN,7} \\ s^{STN,8} \end{pmatrix}$$

1.4.4 From STN to GPi

$$\begin{pmatrix} v_{STN\to GPi}^{GPi,1} \\ v_{STN\to GPi}^{GPi,2} \\ v_{STN\to GPi}^{GPi,3} \\ v_{STN\to GPi}^{GPi,4} \\ v_{STN\to GPi}^{GPi,5} \\ v_{STN\to GPi}^{GPi,6} \\ v_{STN\to GPi}^{GPi,7} \\ v_{STN\to GPi}^{GPi,8} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_M^{STN,1} \\ v_M^{STN,2} \\ v_M^{STN,3} \\ v_M^{STN,4} \\ v_M^{STN,5} \\ v_M^{STN,6} \\ v_M^{STN,7} \\ v_M^{STN,8} \end{pmatrix} \quad \text{(Terman's file)}$$

$$\begin{pmatrix} s_{STN\to GPi}^{GPi,1} \\ s_{STN\to GPi}^{GPi,2} \\ s_{STN\to GPi}^{GPi,3} \\ s_{STN\to GPi}^{GPi,4} \\ s_{STN\to GPi}^{GPi,5} \\ s_{STN\to GPi}^{GPi,6} \\ s_{STN\to GPi}^{GPi,7} \\ s_{STN\to GPi}^{GPi,8} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} s^{STN,1} \\ s^{STN,2} \\ s^{STN,3} \\ s^{STN,4} \\ s^{STN,5} \\ s^{STN,6} \\ s^{STN,7} \\ s^{STN,8} \end{pmatrix} \quad \text{(Our simulations)}$$

1.4.5 From GPi to Thl

$$\begin{pmatrix} s_{GPi\to Thl}^{Thl,1} \\ s_{GPi\to Thl}^{Thl,2} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} s^{GPi,1} \\ s^{GPi,2} \\ s^{GPi,3} \\ s^{GPi,4} \\ s^{GPi,5} \\ s^{GPi,6} \\ s^{GPi,7} \\ s^{GPi,8} \end{pmatrix}$$

1.5 Parameters

1.5.1 STN cells' parameters

- Membrane capacitance:

Parameter	Value
C_M^{STN}	$1\ \frac{pF}{(\mu m)^2}$

- Leak current related parameters:

Parameter	Value
g_L^{STN}	$2.25\,\frac{nS}{(\mu m)^2}$
V_L^{STN}	$-60\,mV$

- Sodium current related parameters:

Parameter	Value
g_{Na}^{STN}	$37.5\,\frac{nS}{(\mu m)^2}$
V_{Na}^{STN}	$55\,mV$
θ_m^{STN}	$-30\,mV$
σ_m^{STN}	$15\,mV$
θ_h^{STN}	$-39\,mV$
σ_h^{STN}	$3.1\,mV$
τ_{h0}^{STN}	$1\,ms$
τ_{h1}^{STN}	$500\,ms$
$\theta_{h\tau}^{STN}$	$-57\,mV$
$\sigma_{h\tau}^{STN}$	$3\,mV$
ϕ_h^{STN}	0.75

- Potasium current related parameters:

Parameter	Value
g_K^{STN}	$45\,\frac{nS}{(\mu m)^2}$
V_K^{STN}	$-80\,mV$
θ_n^{STN}	$-32\,mV$
σ_n^{STN}	$8\,mV$
τ_{n0}^{STN}	$1\,ms$
τ_{n1}^{STN}	$100\,ms$
$\theta_{n\tau}^{STN}$	$-80\,mV$
$\sigma_{n\tau}^{STN}$	$26\,mV$
ϕ_n^{STN}	0.75

- High-threshold calcium current related parameters:

Parameter	Value
g_{Ca}^{STN}	$0.5\,\frac{nS}{(\mu m)^2}$
V_{Ca}^{STN}	$140\,mV$
θ_s^{STN}	$-39\,mV$
σ_s^{STN}	$8\,mV$

- Low-threshold T-type calcium current related parameters:

Parameter	Value
g_T^{STN}	$0.5\,\frac{nS}{(\mu m)^2}$
θ_a^{STN}	$-63\,mV$
σ_a^{STN}	$7.8\,mV$
θ_b^{STN}	$0.25\,mV$
σ_b^{STN}	$0.07\,mV$
θ_r^{STN}	$-67\,mV$
σ_r^{STN}	$2\,mV$
τ_{r0}^{STN}	$7.1\,ms$
τ_{r1}^{STN}	$17.5\,ms$
$\theta_{r\tau}^{STN}$	$68\,mV$
$\sigma_{r\tau}^{STN}$	$2.2\,mV$
ϕ_r^{STN}	0.5

- Calcium-activated voltage-independent "afterhyperpolarization" potasium current related parameters:

Parameter	Value
g_{AHP}^{STN}	$9\,\frac{nS}{(\mu m)^2}$
k_1^{STN}	15
k_{Ca}^{STN}	$22.5\,\frac{pA}{(\mu m)^2}$
$\phi_{x_{Ca}}^{STN}$	0.75
$\epsilon_{x_{Ca}}^{STN}$	$5 \times 10^{-5}\,\frac{(\mu m)^2}{pA}(ms)^{-1}$

- Synaptic state-variables related parameters:

Parameter	Value
α^{STN}	5
β^{STN}	1
θ_g^{STN}	$30\,mV$
θ_{gH}^{STN}	$-39\,mV$
σ_{gH}^{STN}	$8\,mV$

- Afferent synaptic currents related parameters:

Parameter	Value
$g_{GPe\rightarrow STN}$	$0.9\,\frac{nS}{(\mu m)^2}$
$V_{GPe\rightarrow STN}$	$-100\,mV$

- Constant current (used to multiply by the cell index number to create slight imbalance):

Parameter	Value
I_0^{STN}	$2\,\frac{pA}{(\mu m)^2}$

- HFS related parameters (Terman's file):

Parameter	Value
I_{HFS}^{STN}	$0\,\frac{pA}{(\mu m)^2}$ (no HFS) ; $400\,\frac{pA}{(\mu m)^2}$ (HFS)
ω_{HFS}^{STN}	$0.5\,\frac{rad}{ms}$
a_{HFS}^{STN}	0.9

- HFS related parameters (Our simulations):

Parameter	Value
I_{HFS}^{STN}	$0\,\frac{pA}{(\mu m)^2}$ (no HFS) ; $21.43, 42.86, 64.29, 85.71, 107.1, 128.6, 150.0, 171.4,$ $192.9, 214.3, 235.7, 257.1, 278.6, 300.0\,\frac{pA}{(\mu m)^2}$ (HFS)
T_{HFS}^{STN}	$3.00, 6.00, 20.0, 40.0\,ms$
w_{HFS}^{STN}	$0.150, 0.300, 0.600, 0.900\,ms$

1.5.2 GPe and GPi cells' shared parameters

- Membrane capacitance:

Parameter	Value
C_M^{GP}	$1\,\frac{pF}{(\mu m)^2}$

- Leak current related parameters:

Parameter	Value
g_L^{GP}	$0.1\,\frac{nS}{(\mu m)^2}$
V_L^{GP}	$-55\,mV$

- Sodium current related parameters:

Parameter	Value
g_{Na}^{GP}	$120\,\frac{nS}{(\mu m)^2}$
V_{Na}^{GP}	$55\,mV$
θ_m^{GP}	$-37\,mV$
σ_m^{GP}	$10\,mV$
θ_h^{GP}	$-58\,mV$
σ_h^{GP}	$12\,mV$
τ_{h0}^{GP}	$0.05\,ms$
τ_{h1}^{GP}	$0.27\,ms$
$\theta_{h\tau}^{GP}$	$-40\,mV$
$\sigma_{h\tau}^{GP}$	$12\,mV$
ϕ_h^{GP}	0.05

- Potasium current related parameters:

Parameter	Value
g_K^{GP}	$30\,\frac{nS}{(\mu m)^2}$
V_K^{GP}	$-80\,mV$
θ_n^{GP}	$-50\,mV$
σ_n^{GP}	$14\,mV$
τ_{n0}^{GP}	$0.05\,ms$
τ_{n1}^{GP}	$0.27\,ms$
$\theta_{n\tau}^{GP}$	$-40\,mV$
$\sigma_{n\tau}^{GP}$	$12\,mV$
ϕ_n^{GP}	0.05

- High-threshold calcium current related parameters:

Parameter	Value
g_{Ca}^{GP}	$0.1\,\frac{nS}{(\mu m)^2}$
V_{Ca}^{GP}	$120\,mV$
θ_s^{GP}	$-35\,mV$
σ_s^{GP}	$2\,mV$

- Low-threshold T-type calcium current related parameters:

Parameter	Value
g_T^{GP}	$0.5\,\frac{nS}{(\mu m)^2}$
θ_a^{GP}	$-57\,mV$
σ_a^{GP}	$2\,mV$
θ_r^{GP}	$-70\,mV$
σ_r^{GP}	$2\,mV$
τ_r^{GP}	$30\,ms$
ϕ_r^{GP}	1

- Calcium-activated voltage-independent "afterhyperpolarization" potasium current related parameters:

Parameter	Value
g_{AHP}^{GP}	$30\,\frac{nS}{(\mu m)^2}$
k_1^{GP}	30
k_{Ca}^{GP}	$20\,\frac{pA}{(\mu m)^2}$
$\epsilon_{x_{Ca}}^{GP}$	$1 \times 10^{-4}\,\frac{(\mu m)^2}{pA}(ms)^{-1}$

- Synaptic state-variables related parameters:

Parameter	Value
α^{GP}	2
θ_g^{GP}	$20\,mV$
θ_{gH}^{GP}	$-57\,mV$
σ_{gH}^{GP}	$2\,mV$

1.5.3 GPe cells' non-shared parameters

- Synaptic state-variables related parameter:

Parameter	Value
β^{GPe}	0.04

- Afferent synaptic currents related parameters:

Parameter	Value
$g_{GPe\to GPe}$	$1\,\frac{nS}{(\mu m)^2}$ (normal) ; $0\,\frac{nS}{(\mu m)^2}$ (Parkinsonian)
$V_{GPe\to GPe}$	$-80\,mV$
$g_{STN\to GPe}$	$0.3\,\frac{nS}{(\mu m)^2}$
$V_{STN\to GPe}$	$0\,mV$

- Constant current (used to multiply by the cell index number to create slight imbalance):

Parameter	Value
I_0^{GPe}	$0.3\,\frac{pA}{(\mu m)^2}$

- External applied current:

Parameter	Value
I_{app}^{GPe}	$-0.5\,\frac{pA}{(\mu m)^2}$ (normal) ; $-2.3\,\frac{pA}{(\mu m)^2}$ (Parkinsonian)

1.5.4 GPi cells' non-shared parameters

- Synaptic state-variables related parameters:

Parameter	Value
β^{GPi}	0.08

- Afferent synaptic currents related parameters:

Parameter	Value
$g_{STN\to GPi}$	$1\,\frac{nS}{(\mu m)^2}$
$V_{STN\to GPi}$	$0\,mV$
$\theta_{STN\to GPi}$	$0\,mV$

- External applied current:

Parameter	Value
I_{app}^{GPi}	$-1.2\,\frac{pA}{(\mu m)^2}$

1.5.5 Thalamic cells' parameters

- Membrane capacitance:

Parameter	Value
C_M^{Thl}	$1\,\frac{pF}{(\mu m)^2}$

- Leak current related parameters:

Parameter	Value
g_L^{Thl}	$0.05\,\frac{nS}{(\mu m)^2}$
V_L^{Thl}	$-70\,mV$

- Sodium current related parameters:

Parameter	Value
g_{Na}^{Thl}	$3\,\frac{nS}{(\mu m)^2}$
V_{Na}^{Thl}	$50\,mV$
θ_m^{Thl}	$-37\,mV$
σ_m^{Thl}	$7\,mV$
θ_h^{Thl}	$-41\,mV$
σ_h^{Thl}	$4\,mV$
a_{h0}^{Thl}	$0.128\,(ms)^{-1}$
θ_{ah}^{Thl}	$-46\,mV$
σ_{ah}^{Thl}	$18\,mV$
b_{h0}^{Thl}	$4\,(ms)^{-1}$
θ_{bh}^{Thl}	$-23\,mV$
σ_{bh}^{Thl}	$5\,mV$
ϕ_h^{Thl}	1

- Potasium current related parameters:

Parameter	Value
g_K^{Thl}	$5\,\frac{nS}{(\mu m)^2}$
V_K^{Thl}	$-90\,mV$

- Low-threshold T-type calcium current related parameters:

Parameter	Value
g_T^{Thl}	$5\,\frac{nS}{(\mu m)^2}$
V_T^{Thl}	$0\,mV$
θ_p^{Thl}	$-60\,mV$
σ_p^{Thl}	$6.2\,mV$
θ_r^{Thl}	$-84\,mV$ (but see note 1 below)
σ_r^{Thl}	$4\,mV$
τ_{r0}^{Thl}	$28\,ms$ (but see note 2 below)
τ_{r1}^{Thl}	$1\,ms$
$\theta_{r\tau}^{Thl}$	$-25\,mV$
$\sigma_{r\tau}^{Thl}$	$10.5\,mV$ (but see notes 1 and 2 below)
ϕ_r^{Thl}	2.5

Note 1: For the perturbation example we took $\theta_r^{Thl} = -79.8\,mV$ and $\sigma_{r\tau}^{Thl} = 11.025\,mV$.
Note 2: For the simulation of a "fast" T-current we took $\tau_{r0}^{Thl} = 5\,ms$ and $\sigma_{r\tau}^{Thl} = 15\,mV$.

- Afferent synaptic currents related parameters:

Parameter	Value
$g_{GPi\rightarrow Thl}$	$0.15\,\frac{nS}{(\mu m)^2}$
$V_{GPi\rightarrow Thl}$	$-85\,mV$

- Sensorimotor signal related parameters:

Parameter	Value
I_{SM}^{Thl}	$8\,\frac{pA}{(\mu m)^2}$
T_{SM}^{Thl}	$25\,ms$ (Terman's file) ; $50\,ms$ (Our simulations)
w_{SM}^{Thl}	$5\,ms$
d_{SM}^{Thl}	$80\,ms$

- Auxiliary sigmoidal function related parameter:

Parameter	Value
σ_Y	0.001